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## Function spaces with variable exponents

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In this talk we give an overview of results about function spaces with variable exponents. Also we present some recent progress obtained by the author and their collaborators on the theory of variable function spaces including variable Sobolev spaces associated with ultraspherical expansions, variable anisotropic Hardy-Lorentz spaces and variable Hardy spaces on graphs.

The Lebesgue spaces with variable exponent  $L^{p(\cdot)}$  were studied for the first time already 1931 by Orlicz. Later, these spaces were investigated by Nakano (1950), Musielak and Orlicz (1959) and Hudzik (1979) and others in the more general context of generalized Orlicz and modular spaces. An important step in the investigation of variable exponent spaces was the paper of Kovacik and Rakosnik in the early 90's, where many of the basic properties of variable Lebesgue and Sobolev were established. At the turn of the millennium a period of intense study of variable exponent spaces (Lebesgue, Sobolev, Hardy, Triebel-Lizorkin, Besov, Herz spaces,...) started. Apart from theoretical considerations, the function spaces with variable exponents also have interesting applications in fluid dynamics, image processing, partial differential equations and calculus of variations.